

# 結合大型語言模型與數學精熟學習之知識追蹤研究

## Proficiency-Aware Knowledge Tracing: Enhancing Calibration and Explainability via Large Language Models

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**【摘要】** 團隊開發之系統以往多依賴學習者正確率等外顯指標為學生的成果判定, 缺乏深入探究學生對題目語意與認知維度的理解, 導致難以支援精準教學, 為了解決此問題, 本研究提出「精熟感知知識追蹤 (Proficiency-Aware Knowledge Tracing, PKT)」之知識追蹤模型框架, 透過大語言模型 (LLMs), 依據數學題目的架構自動生成題目之認知特徵 (如概念理解、程序流暢性等), 並將其整合至序列式預測模型中。實驗結果顯示引入認知特徵後可降低預測誤差, 提升機率估計之準度。此外, 透過視覺化分析, 模型能區別出學生在不同認知面向的優勢與弱點, 提升預測結果的可解釋性與支援教師教學。

**【關鍵字】** 知識追蹤; 大語言模型; 學習分析

**Abstract:** Previous systems developed by our team largely relied on explicit indicators such as learner accuracy to evaluate student performance. These systems lacked a deep investigation into students' understanding of problem semantics and cognitive dimensions, which made it difficult to support precision instruction. To address this issue, this study proposes a "Proficiency-Aware Knowledge Tracing (PKT)" framework. Leveraging Large Language Models (LLMs), the framework automatically generates cognitive features of problems (such as conceptual understanding and procedural fluency) based on the structure of mathematics problems and integrates them into sequential prediction models. Experimental results indicate that introducing cognitive features reduces prediction errors and improves the accuracy of probability estimation. Furthermore, through visualization analysis, the model can distinguish students' strengths and weaknesses across different cognitive dimensions, enhancing the explainability of prediction results and supporting teacher instruction.

**Keywords:** Knowledge Tracing, Large Language Models, Learning Analytics

### 1. 前言

由於數位化學習環境的普及與人工智慧技術的快速發展, 學生的學習型態與學習資源正逐漸轉變。各式各樣的學習平台、智慧教學系統與 AI 輔助工具相繼出現, 嘗試以更即時、更個別化的方式支援學生學習。然而, 當學習活動大量移轉至數位系統後, 「能否以資料精準推估學生的學習狀況」便成為適性化教學是否可行的關鍵。換言之, 若系統無法可靠地辨識學生當下的理解程度與困難來源, 後續所生成的建議、回饋與介入策略即可能失去幫助, 甚至造成不適切的教學決策。

在既有研究與實務應用中, 許多評量與預測方法仍主要依賴外顯的行為指標, 例如答對率、連續答對次數或作答時間等, 其中以正確性與分數資料的分析最為普遍 (Corbett & Anderson, 1995)。然而, 這類行為資料多半反映的是學生的「學習表現 (Performance)」而非「學習能力 (Competence)」。例如, 高答對率可能來自重複練習某些題型、使用策略性作答或只是短期記憶, 未必能對應到對概念的穩固理解; 相對地, 低答對率也可能源自粗心、閱讀理解不足、情境壓力或題目表徵轉換的認知不足, 並不必然代表學生缺乏核心概念。因此, 若我們在評估學生能力時, 過度仰賴單一或少數表層訊號, 便容易將「會做題」等同於「真懂」, 或將「一時失誤」等同於「不會」, 使得學習狀態的推估與解釋在教育意義上受到限制。

更重要的是, 學習本質並非線性、單向度的累積, 而是多面向能力共同發展、彼此交織的動態歷程; 尤其在結構複雜且高度累進的學科 (如數學) 中, 學習成果往往同時涉及概念理解、程序技能、策略選擇、推理能力與學習態度等層面。Kilpatrick 等人 (2001) 提出的「數學精熟 (Mathematical Proficiency)」框架即指出, 成熟的數學能力由五個互相支撐的面向構成: 概念理解 (conceptual understanding)、程序流暢性 (procedural fluency)、策略能力 (strategic competence)、適性推理 (adaptive reasoning) 以及正向傾向 (productive

disposition)。然而，在真實的資料情境中，研究者與系統能穩定取得的觀測資料，往往仍高度偏向程序流暢性的表徵（例如是否答對、速度是否提升等），對概念理解與推理能力等面向的測量相對不足。此一「理論構念的多維度」與「可觀測資料的單薄化」之間的落差，使得模型即便能有效預測表現，也未必能提供學生足夠具體的能力評量。

因此，在數位化學習情境中，核心問題不僅是「應評量哪些學習面向」，更在於「如何在大規模、即時且可運算的條件下，利用學習系統所蒐集的互動紀錄推論學生的內在知識狀態」。多數學習平台所能取得的資料，通常以題目層級的互動為主（例如正確性、作答時間、提示使用、重作次數等），並以時間序列形式累積成學生的作答歷程。為回應這種以序列互動資料為主的現實，學習分析領域逐漸發展出一條重要技術路線：透過學生作答序列動態估計其知識掌握程度，並據此預測其在後續任務上的表現與學習風險；其中，知識追蹤（Knowledge Tracing, KT）便是具代表性的研究方法之一。

知識追蹤的核心目標在於：根據學生與題目互動所形成的時間序列資料，動態推估學生對特定知識構念（例如技能、概念或知識元件）的掌握程度，並進一步預測學生在下一題或未來題目上的答對機率。近年來，知識追蹤技術的發展逐漸多元化；例如深度知識追蹤

（Deep Knowledge Tracing, DKT）透過循序神經網路建構學生作答序列，在多種資料集上都展現了良好的預測表現（Piech et al., 2015）。然而，現有主流知識追蹤模型的輸入特徵仍普遍受限於二元正確性、作答時間、題目 ID 或技能標籤等「低語意密度」訊號：題目往往被視為離散索引或標籤，而非具備語意結構與認知要求的學習內容。這種特徵設計上的侷限，意味著模型即使能掌握序列中的統計規律，也可能難以理解題目文本與認知負荷如何影響學生表現。當題目之間存在細緻語意差異（例如同一技能標籤下，題目敘述、條件限制、語意陷阱或表徵轉換要求不同），僅以 ID 或標籤表示便可能將不同認知需求的題目壓縮為同一類別，進而使模型難以辨識學生是在文字理解、概念轉換或多步推理等環節受阻。模型的預測效力可能隨任務複雜度提高而下降，且其內部推論也較難轉化為教育者可理解、可採行的教學解釋。換言之，知識追蹤雖在預測表現上持續精進，但在「以學習內容為中心」的特徵表徵與認知意義建模上仍存在瓶頸，限制其在複雜認知任務與真實教學情境中的可用性。

在此背景下，近年大語言模型（Large Language Models, LLMs）的興起，為彌補上述測量落差提供了新的可能性。相較於傳統依賴人工特徵標籤的方式，大語言模型具備更強的語言理解與文本表徵能力，能在較少人工標註的前提下，從題目敘述、學習記錄與文本中萃取更高層次的語意結構，並推測出潛在的認知需求、概念關聯與表徵特徵。大語言模型在教育情境中可用於提取語意特徵、生成內容表徵、輔助判讀學生產出，甚至用於建構與評估學習任務的複雜度資訊，以支援更豐富的學習分析（Kasneci et al., 2023）。因此，本研究也將大語言模型視為「語意特徵生成器」或「認知訊號萃取器」，用以補足傳統資料所欠缺的認知面向資訊。故本研究提出一個結合行為數據與大語言模型語意特徵的預測框架：一方面保留知識追蹤模型在處理序列互動資料上的優勢，另一方面引入由題目文本與學習互動所導出的語意與認知複雜度特徵，使模型得以更貼近多模態的方式進行推論。主要的目標不僅在於提升預測準確率，更在於提升模型輸出對教育實務的可解釋性，使教育者能從預測結果中辨識可能的學生困難來源，並據以設計更精準、更適性化的教學方法與學習支援。

## 2. 文獻探討

### 2.1 學習分析中的預測模型發展

早期學習分析研究多採用邏輯迴歸或決策樹等方法進行預測，但這類模型往往難以掌握學習歷程中關鍵的時間依賴與狀態轉變，故後續研究重心逐步移轉至以序列資料為核心的知識追蹤（Knowledge Tracing, KT）技術。此領域最經典的模型是由 Corbett 與 Anderson

（1995）提出的貝氏知識追蹤（Bayesian Knowledge Tracing, BKT），該模型基於隱藏馬爾可夫模型，透過觀察學生連續的作答反應，來推估學生在特定技能上究竟是處於「已學會」

還是「未學會」的狀態。不過貝氏知識追蹤通常假設每個知識點都是獨立互不影響的，這在知識點彼此緊密關聯的真實學習情境中，往往顯得過於簡化。

為了進一步提升預測能力，學者 Piech 等人 (2015) 將深度學習技術引入此領域，提出了深度知識追蹤 (Deep Knowledge Tracing, DKT)。深度知識追蹤模型擅長從大量的作答數據中自動找出複雜的學習規律，因此在預測學生「下一題是否答對」的準確率上大幅超越了傳統方法，能從大量互動數據中自動學習高維度的潛在表徵 (Latent Representations)，顯著提升了對下一題答對機率的預測準確度。然而，模型的內部運作極為複雜，就像一個黑盒子，教育者往往只能看到預測出的結果，卻無法理解模型是根據什麼理由做出判斷，這使得教師難以完全信賴並依此調整教學。

針對此限制，後續研究致力於引入更具結構性的機制以兼顧效能與可解釋性。例如 Zhang 等人 (2017) 提出的動態鍵值記憶網路 (DKVMN)，利用外部記憶矩陣分離「靜態概念定址」與「動態知識更新」，使模型能更清晰地追蹤特定概念的掌握度變化；而 Ghosh、Heffernan 與 Lan (2020) 的注意力式知識追蹤 (AKT) 則引入注意力機制 (Attention Mechanism)，強化對長距離依賴關係的捕捉，並透過權重分配解析過往互動對當前預測的影響。

整體而言，知識追蹤模型的演進已由早期特徵導向的統計方法，逐步發展為結合深度學習、外部記憶與注意力機制的複雜序列架構。此趨勢反映出該領域不單追求預測精準度的提升，更持續在「序列建模能力」與「教育情境所需的可解釋性」之間尋求最佳平衡，研究者不再單純追求演算法的準確率，而是更強調模型必須具備可解釋性，縮短技術與教學現場的距離，才能真正協助教師進行決策。

## 2.2 大語言模型在預測模型中的應用

隨著自然語言處理技術快速進展，大語言模型 (Large Language Models, LLMs) 逐漸被視為強化教育情境預測模型的重要技術之一，其價值主要體現在「補足語意訊號」與「提升可解釋性」兩個面向。相較於傳統預測模型多依賴正確性、作答時間等結構化數值資料，大語言模型能處理題目敘述、作答內容、解釋文字等非結構化文本，從中萃取潛在語意特徵，並在適當提示下透過思維鏈 (Chain-of-Thought, CoT) 進行逐步推理，使模型得以更細緻地辨識題目背後的邏輯結構與認知需求。基於此能力，近年研究多以「特徵工程」方式將大語言模型納入知識追蹤與學習表現預測。例如 Guo 等人 (2024) 利用大語言模型擷取新題目的屬性與解題需求，以提升缺乏歷史作答資料情境下的預測；另一方面，Yu 等人 (2024) 的結合大語言模型，透過思維鏈推理生成問題描述與知識概念，協助模型更充分捕捉學習序列中的關係。大語言模型也被用來縮小「表層行為指標」與「深層概念理解」之間的落差：透過分析題目文本與解釋內容，模型可反映任務複雜度與語意脈絡的特徵，並自動化標註題目（例如數學精熟度中的概念理解、程序流暢性與策略能力），使預測模型得以引入具理論基礎的認知特徵。當這些特徵被整合進既有預測架構後，模型輸出不再僅停留在「答對機率」的數值預測，而能更接近解釋「學生為何可能未精通」以及其不足可能落在哪一類認知面向，從而提升預測結果對教育者的可理解性。整體而言，大語言模型透過語意理解與推理能力，可以讓模型由單純的資料呈現走向更具教學意義的分析 (Guo et al., 2024; Yu et al., 2024)。

綜整前述文獻可見，當前學習分析中的預測模型雖在技術層面持續精進，但其輸入多仍偏重答對率等表層行為指標，導致模型對數學理解所需的認知細緻度掌握不足，進而與教學現場的診斷需求產生落差。以大語言模型作為橋接工具，將其語意理解能力用於自動分析題目文本與解題需求，生成具教育理論基礎且具認知意義的特徵，例如辨識題目主要考驗的是概念理解、程序操作、策略能力或推理要求，並將這些隱含的認知需求轉化為可輸入預測模型的顯性變項；接著把大語言模型生成之認知特徵整合至既有預測架構中，能在一定程度上呈現出學習內容背後的教育意涵。

因此本研究聚焦於兩個面向：其一為模型效能，是否能更準確地評估學生的學習狀態並預測後續表現；其二為可解釋性，使預測輸出能呈現出具體的成果，而非僅呈現難以轉譯的數值，從而更貼近學生真實的學習歷程並提高教學可用性。

### 3. 研究設計

#### 3.1 資料特徵與項次

基於上述研究意涵，本研究以 Kilpatrick 等人 (2001) 提出之數學精熟框架 (Mathematical Proficiency Framework) 為理論基礎，並將題目認知需求聚焦於三個與教學診斷高度相關且可操作化的精熟度面向：概念理解 (Conceptual Understanding, 簡稱 CU)、程序流暢性 (Procedural Fluency, 簡稱 PF) 與策略能力 (Strategic Competence, 簡稱 SC) 作為特徵。透過大語言模型將上述面向轉譯為可運算之題目特徵，進一步整合至預測模型中，檢驗特徵是否能同時提升預測效能與可解釋性，看看是否能提供教學需求的支援。

#### 3.2 資料集與預處理

為驗證模型在不同預測情境下的有效性，本研究選用兩個小學四年級數學資料集，分別對應序列式預測與門檻式預測任務。資料集 I 為公開大型資料集 XES3G5M，包含約 7,652 個題目 (填充題 6,142 題、選擇題 1,510 題)，蒐集自 18,066 名學生共 5,549,635 筆互動紀錄，並提供階層化知識概念樹與 865 個知識點，可用以建模概念之先備關係與依賴性，適合用於知識追蹤之序列學習情境。資料集 II 來自團隊長期建立的數學線上學習平台「數學島」(Math-Island, 是一個遊戲式數學學習系統)，本次研究主要擷取國小數學四年級上學期內容為資料集，約 1,500 道題目，分別對應 19 個主要概念與 102 個子概念，其中包含約 3,500 名學生、共約 41,500 筆紀錄。本研究將資料集 II 用於精通預測，將精通視為靜態分類任務 (例如是否達到特定精熟門檻)，藉此檢驗在非序列情境中，大語言模型生成之認知特徵是否仍具有判斷能力。

圖 1、XES3G5M 與 Math-Island 資料集示意

| fold | uid   | questions                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               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| 0    | 11066 | 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3751,3752,3753,3754,3755,3756,3757,3758,3759,3760,3761,3762,3763,3764,3765,3766,3767,3768,3769,3770,3771,3772,3773,3774,3775,3776,3777,3778,3779,3780,3781,3782,3783,3784,3785,3786,3787,3788,3789,3790,3791,3792,3793,3794,3795,3796,3797,3798,3799,3800,3801,3802,3803,3804,3805,3806,3807,3808,3809,3810,3811,3812,3813,3814,3815,3816,3817,3818,3819,3820,3821,3822,3823,3824,3825,3826,3827,3828,3829,3830,3831,3832,3833,3834,3835,3836,3837,3838,3839,3840,3841,3842,3843,3844,3845,3846,3847,3848,3849,3850,3851,3852,3853,3854,3855,3856,3857,3858,3859,3860,3861,3862,3863,3864,3865,3866,3867,3868,3869,3870,3871,3872,3873,3874,3875,3876,3877,3878,3879,3880,3881,3882,3883,3884,3885,3886,3887,3888,3889,3890,3891,3892,3893,3894,3895,3896,3897,3898,3899,3900,3901,3902,3903,3904,3905,3906,3907,3908,3909,3910,3911,3912,3913,3914,3915,3916,3917,3918,3919,3920,3921,3922,3923,3924,3925,3926,3927,3928,3929,3930,3931,3932,3933,3934,3935,3936,3937,3938,3939,3940,3941,3942,3943,3944,3945,3946,3947,3948,3949,3950,3951,3952,3953,3954,3955,3956,3957,3958,3959,3960,3961,3962,3963,3964,3965,3966,3967,3968,3969,3970,3971,3972,3973,3974,3975,3976,3977,3978,3979,3980,3981,3982,3983,3984,3985,3986,3987,3988,3989,3990,3991,3992,3993,3994,3995,3996,3997,3998,3999,4000,4001,4002,4003,4004,4005,4006,4007,4008,4009,4010,4011,4012,4013,4014,4015,4016,4017,4018,4019,4020,4021,4022,4023,4024,4025,4026,4027,4028,4029,4030,4031,4032,4033,4034,4035,4036,4037,4038,4039,4040,4041,4042,4043,4044,4045,4046,4047,4048,4049,4050,4051,4052,4053,4054,4055,4056,4057,4058,4059,4060,4061,4062,4063,4064,4065,4066,4067,4068,4069,4070,4071,4072,4073,4074,4075,4076,4077,4078,4079,4080,4081,4082,4083,4084,4085,4086,4087,4088,4089,4090,4091,4092,4093,4094,4095,4096,4097,4098,4099,4100,4101,4102,4103,4104,4105,4106,4107,4108,4109,4110,4111,4112,4113,4114,4115,4116,4117,4118,4119,4120,4121,4122,4123,4124,4125,4126,4127,4128,4129,4130,4131,4132,4133,4134,4135,4136,4137,4138,4139,4140,4141,4142,4143,4144,4145,4146,4147,4148,4149,4150,4151,4152,4153,4154,4155,4156,4157,4158,4159,4160,4161,4162,4163,4164,4165,4166,4167,4168,4169,4170,4171,4172,4173,4174,4175,4176,4177,4178,4179,4180,4181,4182,4183,4184,4185,4186,4187,4188,4189,4190,4191,4192,4193,4194,4195,4196,4197,4198,4199,4200,4201,4202,4203,4204,4205,4206,4207,4208,4209,4210,4211,4212,4213,42 |          |           |            |  |

(2)思維鏈推理：引導大語言模型分析解題所需的認知步驟（如：是否需要定義？是否涉及繁複的計算過程？）。

(3)特徵輸出：依據推理結果，將題目在「概念理解（CU）」、「程序流暢性（PF）」與「策略能力（SC）」三個維度上進行評分（0 到 1 分）。

本研究提出的 PKT 模型採用多維度輸入表示法，以四大維度：(1)內容識別碼(2)答對與否(3)作答時間(4)LLM 生成之精熟度特徵為基礎。在每次學生學習互動時間時，PKT 會將各特徵的嵌入矩陣進行拼接，動態推論學生的能力狀態。具體而言，在時間步  $t$  時，模型將內容識別碼  $c_t$ 、答題結果  $r_t$ 、作答時間  $t_t$  與認知特徵評分  $p_t$  分別映射為嵌入向量，並經由串接操作形成輸入特徵向量  $x_t = [E_c(c_t) \oplus E_r(r_t) \oplus E_t(t_t) \oplus E_p(p_t)]$ 。

在大語言模型的選擇與技術參數設定上，選用 OpenAI GPT-3.5 Turbo 0125 模型進行認知特徵之標註，在思維鏈的提示語設計上，本研究規範模型根據「題目情境、正確答案、簡要解釋、題型、選項」等五項元素進行聯合推理，確保自動生成的認知特徵評分具備一致性。

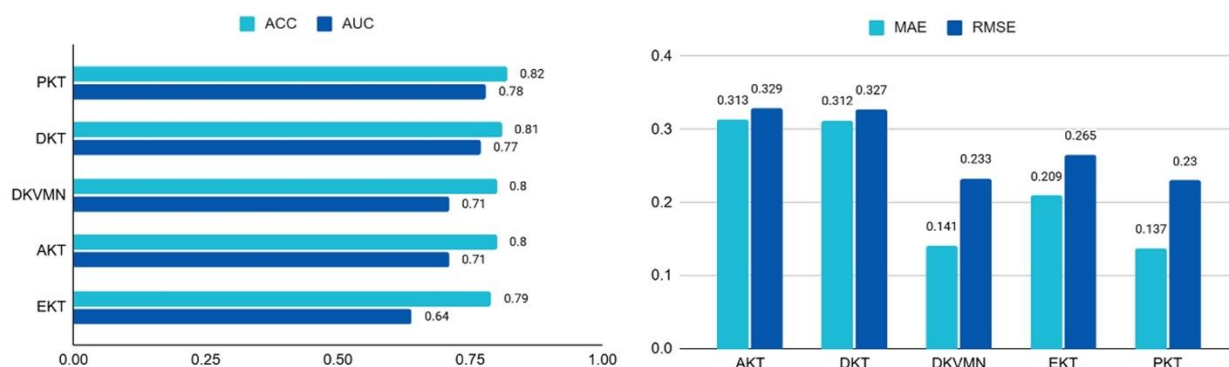
為確保標註品質，本研究採取「90/10 混合驗證策略」。其中 90% 的題目由大語言模型自動標註；而剩餘 10% 的題目樣本（平均抽樣自不同單元與難度）則交由兩位具備學科背景的專家進行雙盲獨立標註。人類專家不僅參與最終品質檢核，更負責前期定義精熟度面向與設計思維鏈提示架構，結果顯示 Cronbach's alpha 一致性係數達 0.85，顯示大語言模型生成之認知特徵可具參考性。同時透過注意力權重與轉換矩陣等視覺化方法，呈現模型在不同精熟度面向上的依據與轉換模式，以提升模型輸出對教學決策的可理解性與可行性（Kilpatrick et al., 2001）。

## 4. 初步研究結果

### 4.1 知識追蹤模型之效能比較

本節以 XES3G5M 資料集進行序列式預測實驗，比較多種知識追蹤模型在學習狀態推估與下一題表現預測上的整體效能，並用以驗證本研究之精熟感知知識追蹤（PKT）架構的有效性。結果顯示，精熟感知知識追蹤在各項指標上皆呈現極佳表現：準確度（ACC）達 0.82，且在預測誤差指標（MAE = 0.137, RMSE = 0.230）表現也很好，顯示其不僅能較正確地分類答對與答錯的判斷，亦能精準地估計模型輸出的答對機率。相較之下，深度知識追蹤（DKT）為次佳（ACC = 0.81），顯示僅依賴行為序列或以概念記憶追蹤的方式，仍可能受限於題目認知需求差異未被充分表徵的問題。本研究也加入精熟度作為特徵並進行分析。結果顯示，引入特徵後模型在誤差指標（MAE 與 RMSE）上有顯著改善，例如深度知識追蹤的 MAE 由 0.312 降至 0.140、RMSE 由 0.327 降至 0.232。顯示大幅修正了機率估計的偏差，有效提升了預測的校準度。儘管部分基準模型在引入特徵後，分類指標的提升幅度相對有限（例如 AKT 模型的 AUC 由 0.71 提升至 0.76），但誤差的顯著降低顯示了模型輸出的機率值更具可靠性。此外，本研究提出的 PKT 模型在 AUC 指標上更達到了 0.78 的整體最佳表現。整體而言，將題目的認知需求納入考量，能使知識追蹤模型從單純的「成績表現預測」，進化為更深層的「知識狀態推估」，並為後續可解釋性分析奠定基礎。

圖 2、多種知識追蹤模型在分類指標（ACC, AUC）與誤差指標（MAE, RMSE）綜合效能比較



## 4.2 精熟度特徵預測之機器學習模型比較

為了驗證「精熟度特徵」是否具有廣泛的適用性，本節利用數學島資料集進行測試，比較了隨機森林（RF）、決策樹（DT）、線性回歸（LR）與 XGBoost 等多種傳統機器學習模型在加入特徵後的表現。結果顯示，XGBoost 的預測效果最佳（RMSE = 0.430，解釋變異量  $R^2 = 0.186$  最高），證明集成式模型能有效處理特徵與學習表現之間複雜的非線性關係。然而，值得注意的是，結構最簡單的線性回歸也有不錯表現，表示本研究生成的特徵即使不依賴高複雜度模型，仍具預測能力。此外，線性模型具有較高可解釋性，有助於研究者與教師理解各項特徵對預測結果的影響，並進一步轉化為教學診斷依據。整體而言，結果顯示精熟度特徵具有跨模型、跨情境的應用潛力，可能成為實用的認知表徵工具。

表 1、各機器學習模型結果

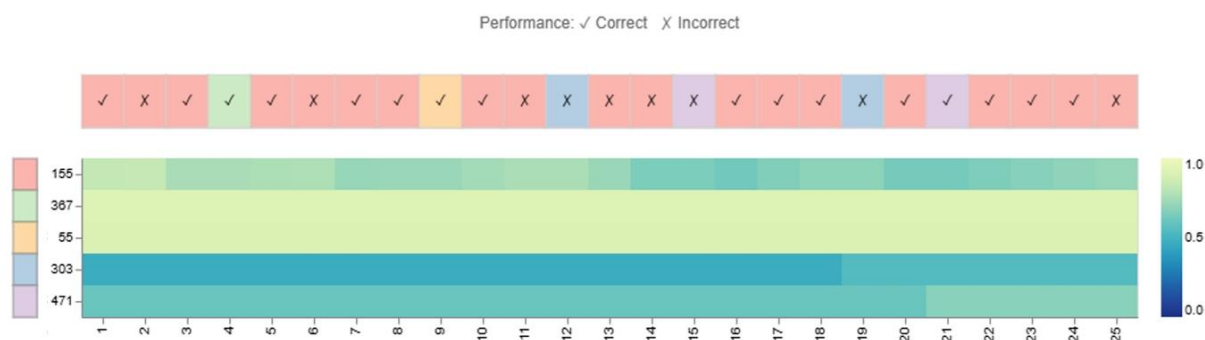
| Model                    | MAE   | RMSE  | R2     | AUC   |
|--------------------------|-------|-------|--------|-------|
| Baseline Mean            | 0.344 | 0.477 | 0.0001 | 0.500 |
| Random Forest Regression | 0.317 | 0.434 | 0.172  | 0.681 |
| Decision Tree Regression | 0.334 | 0.474 | 0.012  | 0.643 |
| Linear Regression        | 0.323 | 0.442 | 0.143  | 0.656 |
| XGBoost Regression       | 0.314 | 0.430 | 0.186  | 0.691 |

## 4.3 視覺化分析

本節透過視覺化分析，探討加入精熟度特徵後，模型在「推論邏輯」與「解釋能力」上的具體改變。首先，在模型推論層面，PKT 模型不再僅僅依賴學生最近答對或答錯的紀錄，而是學會了根據题目的精熟度面向（如概念、程序、策略）來調整預測信心。例如，若學生在「程序流暢性（PF）」題型屢次失敗，模型便會推斷該生具有此面向的「可轉移弱點」，進而調低其同類题目的成功率預測。

其次，在學生學習歷程方面，視覺化結果可以清晰描繪學生的個別差異。以個案為例：編號 UID 為 286401 的學生呈現「懂概念（CU）但算不順（PF）」的特徵；而編號 UID 286403 的學生則表現出跨面向的不穩定與高嘗試次數，暗示其面臨的是整體認知負荷過重或缺乏解題策略。這些發現能協助教師從「看分數」轉向「看歷程」，快速定位問題並提供針對性的教學輔助，提升了預測結果的可行性。

圖 3、視覺化呈現範例



## 5. 結論與未來展望

### 5.1 研究結論與貢獻

本研究提出了一種結合大型語言模型與知識追蹤技術的增強式預測框架（PKT），並以國小數學領域為例，驗證了其在學習狀態推估上的有效性。研究結論歸納如下：

1. 利用 LLM 搭配思維鏈提示，能有效且規模化地將題目比對至數學精熟框架（概念理解、程序流暢性、策略能力），為大量的題庫賦予具認知意義的標籤。
2. 整合認知特徵後的 PKT 模型，在序列預測中顯著降低了預測誤差（RMSE/MAE），使模型更能反映真實的知識狀態。
3. 增強後的模型能協助教育者識別學生在特定認知面向上的優勢與弱點，將純粹的數字呈現轉變為較具解釋性的圖像，看出學生在概念理解、程序流暢性、策略能力的癥結點，幫助教師支援學生的學習。

本研究希望能透過比較不同種的預測模型與方式，尋求可解釋性的大語言模型，幫助到教師更能理解學生的數學學習表現。

### 5.2 研究限制與未來展望

本研究仍存在部分限制，尚待未來研究進一步探討，包含：

1. 本研究目前僅聚焦於國小四年級數學，且主要關注三大精熟面向。未來研究可嘗試擴展至幾何、代數等更高階數學領域，或探索其他學科（如自然科學的探究能力）的適用性。
2. 目前大語言模型對題目的標註是靜態的。然而，同一題目對不同程度的學生可能產生不同的認知負荷（例如對初學者是概念挑戰，對熟練者僅是程序練習）。未來可探索結合學生能力參數的動態特徵生成機制。
3. 隨著題庫規模擴大，如何建立更自動化的品質監控機制將是實務應用的關鍵。
4. 本研究目前的視覺化分析主要作為模型可解釋性的概念驗證，仍偏向個案說明層次。未來將引入「系統化的解釋性評估指標」，例如透過教師可解釋性問卷調查或實際教學現場的訪談，以提供更具參考性的策略支援。

未來工作將持續嘗試不同種的方式，實證此類具備認知解釋力的預測模型，是否能具體改變教師的介入行為並提升學生的長期學習成效。

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